Ferrite filled reflector mode converter for axially corrugated waveguides

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Abstract — **Mode transformation effect is considered for axially tuned corrugated waveguides. Recently mode transformation effect for transmitted fields was introduced by using certain kind of material which couples the propagating fields. In this paper a reflector type mode converter based on ferrite material is proposed. The properties of ferrite can be changed by using static magnetic field. This gives possibilities for tuning to obtain optimal mode transformation in reflection. Properties of transverse fields are considered in reflection.**

I. Introduction

Propagating waves inside a tuned corrugated circular waveguide filled with gyrotropic material, e.g., ferrite, is considered. The corrugation is in axial direction and the depth of the corrugation is a quarter wave length forming a boundary condition equal to hard surface [1]-[3]. It is assumed that the material is slightly anisotropic and gyrotropic. Also it is assumed that the material is lossless. The small anisotropy and gyrotropy can be achieved at certain frequency range with a proper axial magnetic field strength. The eigenwaves are almost circularly polarized and are propagating almost with the same propagation factor. The small difference in propagation factors affects to the polarization of the propagating field which is seen as a mode transformation between TE and TM modes.

Time harmonic fields (depending on t as $e^{j\omega t}$) are considered and the propagating fields depend on z as $e^{-j\beta z}$, where the propagation factor β is a real number. The waveguide is filled with ferrite material biased with static magnetic field which is in z direction. Only a small gyrotropy is reguired for mode transformation effect. Small anisotropy and small gyrotropy also simplifies the theoretical analysis and in practical point of view makes the matching easily realizable.

II. Fields in ferrite filled hard surface waveguide

For ferrite the constitutive relations are

$$\mathbf{D} = \epsilon\mathbf{E}\ , \quad \mathbf{B} = [\mu_t\overline{\overline{I}}_t + \mu_z\mathbf{u}_z\mathbf{u}_z + j\mu_g\mathbf{u}_z\times\overline{\overline{I}}]\cdot\mathbf{H} \quad (1)$$

where the material parameters are [4]

$$\mu_t = \mu_o(1+\frac{\omega_o\omega_m}{\omega_o^2-\omega^2}), \quad \mu_z = \mu_o, \quad \mu_g = -\mu_o\frac{\omega\omega_m}{\omega_o^2-\omega^2} \quad (2)$$

where $\omega_o = \gamma B_o$ is the Larmor precession frequency (B_o is the stength of the static magnetic flux density in z direction), $\omega_m = \mu_o\gamma M_s$, γ is the gyromagnetic ratio and M_s is the saturation magnetization [4]. Geometry of the problem is shown in Figure 1.

The Maxwell equations

$$\nabla\times\mathbf{E} = -j\omega\mathbf{B}, \quad \nabla\times\mathbf{H} = j\omega\mathbf{D} \quad (3)$$

are written in terms of axial and transverse components

$$\mathbf{E} = \mathbf{e} + E_z\mathbf{u}_z, \quad \mathbf{H} = \mathbf{h} + H_z\mathbf{u}_z \quad (4)$$

and the transverse parts are eliminated. Also wavefield decomposition for axial field components

$$E_{z\pm} = \frac{1}{2}[E_z \mp j\eta H_z] \quad (5)$$

is used [5]. Assuming small anisotropy and small gyrotropy , finally, the Maxwell equations reduce to Helmholtz equation for axial field components

$$\nabla_t^2 E_{z\pm} + \left[\frac{k^2-\beta_\pm^2}{1\mp\frac{\beta_\pm k}{k^2-\beta_\pm^2}\frac{\mu_g}{\mu}}\right]E_{z\pm} = 0 \quad (6)$$

or

$$[\nabla_t^2 + k_{c\pm}^2]E_{z\pm} = 0 \quad (7)$$

which is solved. In the Helmholtz equation ∇_t is the transverse part of the ∇ operator and $k_{c\pm}$ is the cut-off number. The boundary condition for hard surface waveguide [2]

$$E_z = 0, \quad H_z = 0 \quad (8)$$

results to eigenvalue equation from which the cut-off number and finally the propagation factors for eigenwaves are obtained. In cylindrical geometry the cut-off number $k_{c\pm} = k_c = p_{ns}/a$ where p_{ns} are the zeros of the Bessel function and a is the radius of the hard surface waveguide. In rectangular geometry, the

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cut-off number $k_c = \sqrt{k_{cx}^2 + k_{cy}^2}$ with $k_{cx} = m\pi/a$ and $k_{cy} = n\pi/b$ where a and b are the width and the height of the rectangular waveguide, respectively. For a rectangular waveguide the mode indices are $m,\ n = 1,\ 2, \ldots$

Waves are propagating in z direction. For small anisotropy and gyrotropy the propagation factors for the eigenwaves reduce to

$$\beta_{\pm} = \beta \mp \frac{k\mu_g}{2\mu} \tag{9}$$

where $\beta = \sqrt{k^2 - k_c^2}$. For example, with the value for ferrite, $\epsilon_r = 8$, $\omega_m = 2\pi \cdot 2.8$ GHz and with the static magnetic flux density $B_o = 0.03$ T, the ratio $\mu_t/\mu_z = \mu_t/\mu = 0.998 \approx 1$ and $\mu_g/\mu = 0.08$ which results for a circular waveguide with mode $p_{01} = 2.405$ and radius $a = 1.7$ mm at frequency 35 GHz to the value of the propagation factors $\beta_{\pm} = 1.51 \mp 0.08$ 1/mm.

The axial partial fields are in cylindrical waveguide

$$E_{z\pm}(\rho, \varphi, z) = A_{n\pm} J_n(k_c\rho) e^{jn\varphi} e^{-j\beta_{\pm} z} \tag{10}$$

and in rectangular waveguide

$$E_{z\pm}(x, y, z) = A_{n\pm} \sin k_{cx}x \sin k_{cy}y \ e^{-j\beta_{\pm} z} \tag{11}$$

Denoting the function depending on the transverse coordinates by F (function $F = J_n(k_c\rho)e^{jn\varphi}$ or $F = \sin k_{cx}x \sin k_{cy}y$) the total axial field components are obtained in the form

$$E_z = [A_{n+}e^{-j\beta_+ z} + A_{n-}e^{-j\beta_- z}]F \tag{12}$$

and

$$H_z = \frac{j}{\eta}[A_{n+}e^{-j\beta_+ z} - A_{n-}e^{-j\beta_- z}]F \tag{13}$$

The transverse electric and magnetic fields are [4]

$$\mathbf{e} = -j\frac{\beta}{k_c^2}\nabla_t E_z + j\frac{k\eta}{k_c^2}\mathbf{u}_z \times \nabla_t H_z \tag{14}$$

$$\mathbf{h} = -j\frac{\beta}{k_c^2}\nabla_t H_z - j\frac{k}{k_c^2\eta}\mathbf{u}_z \times \nabla_t E_z \tag{15}$$

Instead of using these transverse fields in slightly gyrotropic medium it is practically use elliptically polarized vectors [6]

$$\mathbf{e}_{\pm} = \left[-\frac{j\beta}{k_c^2}\nabla_t \mp \frac{k}{k_c^2}\mathbf{u}_z \times \nabla_t\right] E_{z\pm} \tag{16}$$

where the corresponding + and − fields are propagating with different propagation factors β_+ and β_-, respectively. The total transverse electric field is

$$\mathbf{e} = \mathbf{e}_+ + \mathbf{e}_- \tag{17}$$

and the transverse magnetic field is

$$\mathbf{h} = \frac{j}{\eta}[\mathbf{e}_+ - \mathbf{e}_-] \tag{18}$$

In the total field the partial + and − fields suffer a phase shift relative to each other and the polarization of the total field is changed.

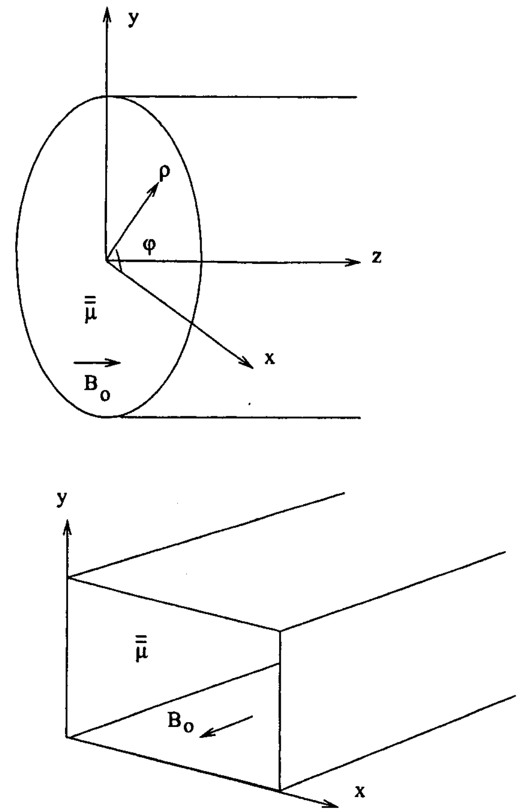

Fig. 1. Circular or rectangular hard surface waveguide filled with ferrite. The static magnetic field is in axial direction

III. Reflected fields from metal terminated hard surface waveguide

The transverse fields are given in terms of + and − eigenwaves

$$\mathbf{e}_{\pm} = \left[-\frac{j\beta}{k_c^2}\nabla_t \mp \frac{k}{k_c^2}\mathbf{u}_z \times \nabla_t\right] A_{n\pm} F e^{-j\beta_{\pm} z} \tag{19}$$

The following two transverse vectors perpendicular to each other are denoted:

$$\mathbf{u} = \nabla_t F(\rho, \varphi), \quad \mathbf{w} = \mathbf{u}_z \times \mathbf{u} \tag{20}$$

Generally these vectors are elliptically polarized but they can be also linearly polarized if instead of $e^{jn\varphi}$ the respective sine or cosine functions are used. In rectangular geometry $\mathbf{u}$ and $\mathbf{w}$ are real vectors. The total transverse electric field propagating in z direction

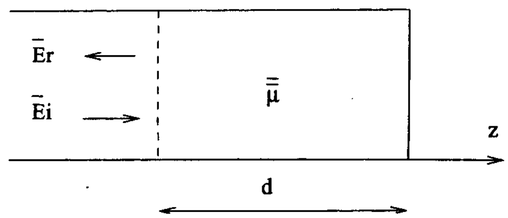

Fig. 2. Ferrite filled hard surface waveguide terminated by metal conductor

(incident field) can be written as

$$\mathbf{e}^i(z) = -\frac{j\beta}{k_c^2}(A_+e^{-j\beta_+z} + A_-e^{-j\beta_-z})\mathbf{u}$$

$$-\frac{k}{k_c^2}(A_+e^{-j\beta_+z} - A_-e^{-j\beta_-z})\mathbf{w} \qquad (21)$$

The reflected transverse field is obtained by changing the sign of the propagation factor β. The sign of the $k\mu_g/\mu$ in the propagation factor $\beta_\pm$ is not changed. The static magnetic field is now reverse to the propagation direction. Hence, the reflected transverse electric field is

$$\mathbf{e}^r(z) = \frac{j\beta}{k_c^2}(A_+^r e^{j\beta_-z} + A_-^r e^{j\beta_+z})\mathbf{u}$$

$$-\frac{k}{k_c^2}(A_+^r e^{j\beta_-z} - A_-^r e^{j\beta_+z})\mathbf{w} \qquad (22)$$

The reflection from conductor terminated waveguide section is illustrated in Figure 2. On the metal boundary at $z = 0$, the total transverse electric field should vanish

$$\mathbf{e}^i + \mathbf{e}^r = 0 \qquad (23)$$

which written separately for $\mathbf{u}$ and $\mathbf{w}$ components results in

$$A_+^r = A_-, \quad A_-^r = A_+ \qquad (24)$$

Now, the reflected transverse electric and magnetic fields can be written by using (22), (24) and (18). The total reflected field consists of the transverse and axial components. The axial field components are considered in detail in the following section.

IV. Mode transformation

The axial field components of the incident wave are

$$E_z^i = [A_+e^{-j\beta_+z} + A_-e^{-j\beta_-z}]F \qquad (25)$$

and

$$H_z^i = \frac{j}{\eta}[A_+e^{-j\beta_+z} - A_-e^{-j\beta_-z}]F \qquad (26)$$

After reflection from conductor terminated waveguide, the reflected axial field components are (using the conditions followed by the boundary condition for conductor (24))

$$E_z^r = [A_+e^{j\beta_+z} + A_-e^{j\beta_-z}]F \qquad (27)$$

and

$$H_z^r = -\frac{j}{\eta}[A_+e^{j\beta_+z} - A_-e^{j\beta_-z}]F \qquad (28)$$

Let us take a section of length d of ferrite filled waveguide terminated by conductor. Comparing the incident axial field components and the reflected ones at $z = -d$, the following relation between these field components is obtained

$$\begin{bmatrix} E_z^r \\ \eta H_z^r \end{bmatrix} = \begin{bmatrix} R_{EE} & R_{EH} \\ R_{HE} & R_{HH} \end{bmatrix} \begin{bmatrix} E_z^i \\ \eta H_z^i \end{bmatrix}$$

$$= e^{-j(\beta_+ + \beta_-)d} \times$$

$$\begin{bmatrix} \cos(\beta_+ - \beta_-)d & -\sin(\beta_+ - \beta_-)d \\ -\sin(\beta_+ - \beta_-)d & -\cos(\beta_+ - \beta_-)d \end{bmatrix} \begin{bmatrix} E_z^i \\ \eta H_z^i \end{bmatrix} \qquad (29)$$

Finally, using (9) the reflected axial field components are written in terms of the incident axial field components as

$$E_z^r = e^{-j2\beta d}[E_z^i \cos(k\frac{\mu_g}{\mu}d) + \eta H_z^i \sin(k\frac{\mu_g}{\mu}d)] \qquad (30)$$

and

$$H_z^r = e^{-j2\beta d}[\frac{E_z^i}{\eta}\sin(k\frac{\mu_g}{\mu}d) - H_z^i \cos(k\frac{\mu_g}{\mu}d)] \qquad (31)$$

It is seen that by choosing the length d such that

$$k\frac{\mu_g}{\mu}d = \frac{\pi}{2}, \qquad (32)$$

for example, the original TM field is changed to TE field in reflection. The absolute values of the reflection coupling coefficients are illustrated as a function of d in matched case in Figure 3 with the material parameter values given in the second section. The required lenght for optimal mode conversion is $d = 9.5$ mm as seen in Figure 3. Similar kind of mode transformer realized by chiral material for transmitted fields was given in [6]. Assuming that there is no reflection at the interface between isotropic and ferrite filled waveguide section the incident TM_{ns} mode is transformed to TE_{ns} mode or vice versa in reflection. The matching at the interface can be made by conventional matching techniques.

V. Transverse fields

Considering the transverse electric field in reflection the relation between the incident and reflected transverse fields are obtained. Eliminating the coefficients $A_\pm$ in (21) and using (22) and (24) the following relation between the reflected and incident transverse fields is found at $z = -d$

$$\mathbf{e}^r = e^{-j2\beta d}\Big[-\cos(k\frac{\mu_g}{\mu}d)\mathbf{uu} - \frac{\beta}{k}\sin(k\frac{\mu_g}{\mu}d)\mathbf{uw} + \frac{k}{\beta}\sin(k\frac{\mu_g}{\mu}d)\mathbf{wu} - \cos(k\frac{\mu_g}{\mu}d)\mathbf{ww}\Big]\cdot\mathbf{e}^i \quad (33)$$

This is the reflecion dyadic for the transverse electric field. All the power is reflected but as is seen from the expression of the dyadic inside the square brackets, the reflection dyadic rotates the orientation of the incident transverse electric field vector. At high frequancy $\beta \to k$ this dyadic approaches to a unit rotator dyadic.

Similarly the relation between the incident and reflected transverse magnetic fields are obtained by using (18). The incident transverse magnetic field is

$$\mathbf{h}^i = \frac{j}{\eta}\Big[-\frac{j\beta}{k_c^2}(A_+e^{-j\beta_+z} - A_-e^{-j\beta_-z})\mathbf{u} - \frac{k}{k_c^2}(A_+e^{-j\beta_+z} + A_-e^{-j\beta_-z})\mathbf{w}\Big] \quad (34)$$

and the reflected transverse magnetic field is after using (24)

$$\mathbf{h}^r = \frac{j}{\eta}\Big[-\frac{j\beta}{k_c^2}(A_+e^{j\beta_+z} - A_-e^{j\beta_-z})\mathbf{u} - \frac{k}{k_c^2}(A_+e^{j\beta_+z} + A_-e^{j\beta_-z})\mathbf{w}\Big] \quad (35)$$

Eliminating the coefficients $A_\pm$ the reflected field is

$$\mathbf{h}^r = e^{-j2\beta d}\Big[\cos(k\frac{\mu_g}{\mu}d)\mathbf{uu} + \frac{\beta}{k}\sin(k\frac{\mu_g}{\mu}d)\mathbf{uw} + \frac{k}{\beta}\sin(k\frac{\mu_g}{\mu}d)\mathbf{wu} - \cos(k\frac{\mu_g}{\mu}d)\mathbf{ww}\Big]\cdot\mathbf{h}^i \quad (36)$$

It is seen that the direction of the magnetic field vector is changed in reflection but it is not rotated as was the case for transverse electric field vector.

VI. Conclusion

The ferrite filled hard surface waveguide of a proper length which is terminated by metal is shown to work as a reflector mode converter between TE and TM fields. Conditions for optimal mode conversion are given. The most effective conversion in reflection is obtained when the wave impedances are equal in isotropic and ferrite waveguide sections. This is possible by taking a proper isotropic material or using matching. Also small changes in material parameters can be achieved by tuning. This kind of dual reflector mode transformer may have applications in waveguide reflectors, resonators or antenna feed devices.

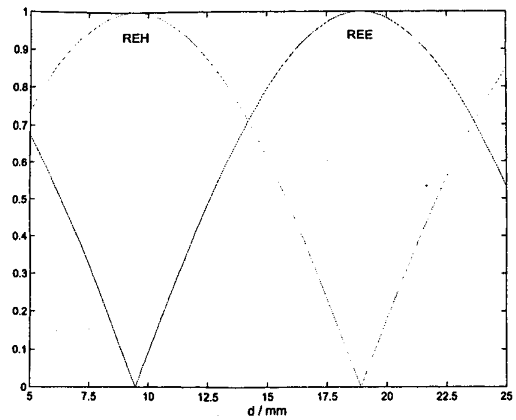

Fig. 3. The absolute value of the reflected coupling coefficients, the permittivity of the isotropic waveguide $\epsilon_r = 8$ (same as in ferrite)

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